Suppose that the drying time for a certain type of paint under specified test conditions is known to be normally distributed with mean 75 minutes and standard deviation 5 minutes. Suppose that chemists have devised a new additive that is hoped will reduce the mean drying time (without changing the standard deviation). Suppose that a test is conducted to measure the drying time for a test specimen, and suppose that company executives decide that they will be convinced that the additive is effective only if the drying time on this specimen is less than 70 minutes.

a) If the additive actually has no effect at all on the drying time, what is the probability that the company executives will mistakenly conclude that it is effective? Include a shaded sketch with your calculation, and also report the relevant $z$-score. \[(\text{Hint: You can do this by hand with the normal probability table, or you can use the Normal Probability Calculator applet, or you can use Minitab: Graph> Probability Distribution Plot> Normal.)}\]

Now suppose that the additive really is effective and that it reduces the mean drying time to 64 minutes, without changing the standard deviation of 5 minutes.

b) Produce a sketch of the two normal curves on the same scale.

c) What is the probability that this test will fail to convince the executives that the additive is effective, even though it actually is (because the mean drying time is now actually 64 minutes)? Again report the relevant $z$-score with your answer.

d) If you want alter the cut-off value from 70 in order to reduce the error probability in a) (meaning the probability that the executives mistakenly conclude that the additive is effective when it really is not) to .025, what cut-off value should you choose? Justify your answer.

e) Using this new cut-off value from d), what is the probability that that the test will fail to convince the executives that the additive is effective, even when it actually is (i.e., when it actually reduces the mean drying time to 64 minutes)?

f) How does the probability in e) compare to that in c)? Explain why this makes sense.

Now suppose that the additive reduces the mean drying time to 64 minutes and also changes the standard deviation to 3 minutes.

g) Produce a sketch of the two normal curves (the original curve assuming that the additive is not effective, and this new one) on the same scale.
h) Still using the cut-off from d), what is the probability that the test will fail to convince the executives that the additive is effective, even when it actually is (i.e., when it actually reduces the mean drying time to 64 minutes)?

i) How does the probability in h) compare to that in e)? Explain why this makes sense.