STAT 313/513 Assignment 8
Due Thursday, 5/12

Testing polynomials, p. 206
See textbook for solutions

Partitioning a sum of squares, p. 207

1. Interaction plot and table of means

> with(barley, interaction.plot(space, variety, yield))

![Interaction plot](image)

> with(barley, tapply(yield, list(space, variety), mean))

<table>
<thead>
<tr>
<th>Space</th>
<th>Variety 1</th>
<th>Variety 2</th>
<th>Variety 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>47.50</td>
<td>50.75</td>
<td>55.75</td>
</tr>
<tr>
<td>2</td>
<td>62.25</td>
<td>58.50</td>
<td>52.25</td>
</tr>
<tr>
<td>3</td>
<td>56.00</td>
<td>64.25</td>
<td>73.00</td>
</tr>
</tbody>
</table>

> m1 = lm(yield ~ block + factor(space)*variety, data = barley)
> anova(m1)

```
Analysis of Variance Table
Response: yield
             Df  Sum Sq Mean Sq F value   Pr(>F)
block         3 255.64  85.21  4.8221 0.00912 **
factor(space) 2 155.06  77.53  4.3872 0.02377 *
variety       2 1027.39 513.69 29.0694 3.872e-07 ***
factor(space):variety  4 765.44 191.36 10.8289 3.679e-05 ***
Residuals     24 424.11  17.67
```
2. From the above model we note that there is a significant space variety interaction \(F_{4, 24} = 10.82, p = 0.00003\) and thus both variety and spacing significantly affect mean yield. Because of the significant interaction, we need to use caution at interpreting main effects.

3. Quadratic model:

\[
\text{> m1 = lm(yield ~ block + space\times variety + I(space^2)\times variety, data = barley)}
\]

\[
\text{> anova(m1)}
\]

Analysis of Variance Table

<table>
<thead>
<tr>
<th>factor</th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>block</td>
<td>3</td>
<td>255.64</td>
<td>85.21</td>
<td>4.8221</td>
<td>0.00912 **</td>
</tr>
<tr>
<td>space</td>
<td>1</td>
<td>155.04</td>
<td>155.04</td>
<td>8.7736</td>
<td>0.00679 **</td>
</tr>
<tr>
<td>variety</td>
<td>2</td>
<td>1027.39</td>
<td>513.69</td>
<td>29.0694</td>
<td>3.872e-07 ***</td>
</tr>
<tr>
<td>I(space^2)</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.0008</td>
<td>0.97787</td>
</tr>
<tr>
<td>space:variety</td>
<td>2</td>
<td>759.08</td>
<td>379.54</td>
<td>21.4779</td>
<td>4.499e-06 ***</td>
</tr>
<tr>
<td>variety:I(space^2)</td>
<td>2</td>
<td>6.36</td>
<td>3.18</td>
<td>0.1800</td>
<td>0.83640</td>
</tr>
<tr>
<td>Residuals</td>
<td>24</td>
<td>424.11</td>
<td>17.67</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. We note that if we add the df and SS for the interaction terms, they add to the same values in the table listed in part 1 (2+2 = 4, and (roughly) 759+6 = 765).

5. The polynomial decomposition shows that the nature of the interaction is linear with no evidence of a quadratic relationship (linear \(F_{2, 24} = 21.47, p < 0.0001\); quadratic \(F_{2, 24} = 0.18, p = 0.836\)). Graphically we see that varieties 1 and 3 show increasing yields with spacing while variety 2 shows a decreasing yield.

6. With the first analysis we could only say there was a significant interaction, but could not distinguish the nature of the interaction. With the quadratic model we can describe that the nature of the interaction is linear and not quadratic (well, at least no evidence that it’s quadratic).