My prediction for the probability of finding at least two people with matching birthdays among the 34 people in lab was 10%. I also predicted that the smallest of people necessary to exceed a probability of .5 was 365/2 or 182. For a probability exceeding .9, I predicted 328 would be necessary.

To find the probability that that there is at least one birthday match among a group of 3 people, I used the complement rule on the probability of no matching birthdays. This resulted in a probability of .0082. Using the same method, I computed the probability of at least one match in a group of 8 people to be .07433.

The complement rule was used once again to find the probability of a match in a group of \( n \) people. The equation is shown below.

\[
P = 1 - \frac{365!}{(365 - n)! 365^n} \]

The values of \( n \) for which the probability is equal to one are 366 and above. This is because there has to be at least one match given there are only 365 original birthday options.

Excel was used to calculate the probabilities for an increasing number of \( n \). The excel formulas are shown below.

\[
P(\text{no match}) = \frac{(365-A3+1)*B2}{365} \quad P(\text{match}) = 1-B3
\]

The probabilities of selected values are shown in the table below.

<table>
<thead>
<tr>
<th>( n )</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
<th>75</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(\text{match}) )</td>
<td>0.0082</td>
<td>0.0271</td>
<td>0.1169</td>
<td>0.4114</td>
<td>0.7063</td>
<td>0.9704</td>
<td>0.9997</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

The smallest value of \( n \) for which the probability exceeds .5 is 23 people. The resulting probability is \( P = .5073 \). To exceed a probability of .9, 41 people are needed. For 41 people, the probability is .9031.

The complete graph of probabilities up to 100 people is shown in the graph below.
This graph shows a steady increase and then a leveling off near a probability of 1 for values above 60.

Now, to change the focus to matching a specific birthday among a group. The probability that at least one person in a group of \( n \) shares Johnny Carson’s birthday can be given by the equation:

\[
P(\text{match}) = 1 - \frac{364^n}{365^n}.
\]

The results of this equation produce the following values.

<table>
<thead>
<tr>
<th>( n )</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(\text{specific match}) )</td>
<td>0.0082</td>
<td>0.0136</td>
<td>0.0271</td>
<td>0.0534</td>
<td>0.0790</td>
<td>0.1282</td>
<td>0.1860</td>
<td>0.2399</td>
<td>0.6663</td>
</tr>
</tbody>
</table>

The graph comparing the values for any match and a specific match is shown below.
As shown in the graph, the specific match has a much more gradual curve, and is barely over .6 when the probability for any match has reached 100%.

The smallest value of $n$ for which the probability of matching a specific day exceeds .2 is 82 people, with a corresponding probability of .20146. To exceed .5, 253 people are needed for a probability of .50048, and to exceed .9, 840 people are needed to obtain a probability of .90019. The only way to get the probability to exactly equal one is theoretically, and requires setting $n$ equal to infinity. This is because there is no way to guarantee that a specific day will be repeated when all others can still be freely chosen.

The drastic differences in the probabilities for these two situations are caused by the nature of each question, and how it affects the available options for matches. When matching any birthday, the field of values producing no match quickly shrinks because every new addition has the potential to match an existing value. It also means a match is guaranteed with 366 people. When considering a specific match, additional people duplicating dates has no effect, so a far greater number of people are required. This also means that no number of people will guarantee a match with the specific date you are interested in.