Stat 321 - Formulas

Do not write on this page and return at the end of the exam.

De Morgan's Laws:
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

Combinations
$$C_{k,n} = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Permutations
$$P_{k,n} = \frac{n!}{(n-k)!}$$

General Addition Rule
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Complement Rule
$P(A') = 1-P(A)$

Conditional Probability
$P(A|B) = P(A \cap B)/P(B)$

Multiplication Rule
$P(A \cap B)=P(A|B)P(B) = P(B|A)P(A)$

Law of Total Probability
$P(B) = \Sigma P(B|A_i)P(A_i)$

Bayes' Theorem
$$P(A_k|B) = \frac{P(A_k \cap B)}{P(B)} = \frac{P(B|A_k)P(A_k)}{\sum_i P(B|A_i)P(A_i)}$$

Cumulative distribution function
Discrete: $F(x) = \Sigma p(y)$ for all $y \le x$
Continuous: $F(x) = \int_{-\infty}^{x} f(t)dt$

Expected Value:
Discrete: $\mu = E(X) = \Sigma x p(x)$ for all x
Expected Value of a function:
$E(h(X)) = \Sigma h(x)\, p(x)$

Continuous: $E(X) = \int x\, f(x)\, dx$ for all x
Expected Value of a function;
$E(h(X)) = \int h(x)\, f(x)\, dx$

Rules of Expected Value:
$E(aX+b) = aE(X) + b$

Variance:
Discrete: $V(X) = \Sigma(x-E(X))^2 p(x)$ for all x

Continuous: $V(X) = E[(X-\mu)^2]$

Shortcut formula: $E(X^2)-[E(X)]^2$

Rules of Variance:
$Var(aX+b)=a^2\, Var(X)$

Binomial Random Variable

P(X=x) = $\binom{n}{x} p^x (1-p)^{n-x}$, x=0,1,…n

E(X) = np V(X) = np(1-p)

Hypergeometric Random Variable

P(X=x) = $\dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$,

for max(0, n-N+M)≤ x ≤min(n, M)

E(X) = n M/N

Negative Binomial Random Variable

P(X=x) = $\binom{x+r-1}{r-1} p^r (1-p)^x$, x=0, 1,…

E(X) = r(1-p)/p V(X) = r(1-p)/p^2

Geometric Random Variable

P(X=x) = $(1-p)^x p$, x=0, 1, …

E(X) = (1-p)/p V(X)=(1-p)/p^2

Poisson Random Variable

P(X=x) = $e^{-\lambda}\lambda^x/x!$, x=0, 1, …

E(X) = λ V(X)=λ

Normal Random Variable

$f(x;\, \mu, \sigma)=\dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$, $-\infty \le x \le \infty$

E(X)=μ V(X)=σ^2

F(x; μ, σ)=$\Phi\left(\dfrac{x-\mu}{\sigma}\right)$

Note:

Γ(n)=(n-1)!

Γ(.5) = $\sqrt{\pi}$

Gamma Random Variable

$f(x;\, \alpha, \beta) = \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$, $x \ge 0$

E(X) = αβ V(X) = $\alpha\beta^2$

P(X ≤ x) = F(x/β;α)

Exponential Random Variable

$f(x;\, \lambda) = \lambda e^{-\lambda x}$ for $x \ge 0$

E(X) = 1/λ V(X) = $1/\lambda^2$

F(x; λ) = $1-e^{-\lambda x}$ for $x \ge 0$

Weibull Random Variable

$f(x;\, \alpha,\beta) = \dfrac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}$ $x \ge 0$

E(X) = βΓ(1+1/α)

F(x; α, β) = $1 - e^{-(x/\beta)^\alpha}$

Lognormal Random Variable

$f(x;\, \mu, \sigma) = \dfrac{1}{x\sigma\sqrt{2\pi}} e^{-[\ln(x)-\mu]^2/(2\sigma^2)}$ $x \ge 0$

E(X)= $e^{\mu+\sigma^2/2}$

F(x; μ, σ)=$\Phi\left(\dfrac{\ln(x)-\mu}{\sigma}\right)$

Uniform Random Variable

$f(x;\, A, B) = \dfrac{1}{B-A}$ for A≤x≤B

E(X)=(A+B)/2

V(X)=$(B-A)^2/12$

Beta Random Variable

$f(x;\, \alpha, \beta, A, B)$ =

$$\frac{1}{B-A}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\left(\frac{x-A}{B-A}\right)^{\alpha-1}\left(\frac{B-x}{B-A}\right)^{\beta-1} \text{ for } A \le x \le B$$

E(X) = A+(B-A)$\dfrac{\alpha}{\alpha+\beta}$

V(X)=$\dfrac{(B-A)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

Rules of Thumb:

- The binomial distribution approximates the hypergeometric distribution when N>20n
- The Poisson distribution approximates the binomial when n≥100, p≤.01 and np≤20.
- The normal distribution approximates the binomial when np≥10 and n(1-p)≥10.

Stat 321 - Formulas

Rules of Expected Value

$E(aX + b) = aE(X) + b$

Rules of Variance

$V(aX + b) = a^2V(X)$

Rules for Linear Combinations of random variables

$E(a_1X_1 + a_2X_2 + \ldots + a_nX_n) = a_1E(X_1) + a_2E(X_2) + \ldots + a_nE(X_n)$

If $X_1,\ldots X_n$ are independent, $V(a_1X_1 + a_2X_2 + \ldots + a_nX_n) = a_1^2V(X_1) + a_2^2V(X_2) + \ldots + a_n^2V(X_n)$

If $X_1,\ldots X_n$ are independent, normally distribution random variables, then any linear combination of the X_i's also has a normal distribution.

Proposition

Let $X_1, \ldots, X_n$ be a random sample from a *normal* distribution with mean μ and variance σ^2. Then for any n, $\overline{X}$ is normally distributed with $\mu_{\overline{X}} = \mu$ and $\sigma^2_{\overline{X}} = \sigma^2/n$, and $T_0 = \Sigma X_i$ also has a normal distribution with $\mu_{T_0} = n\mu$ and $\sigma^2_{T_0} = n\sigma^2$.

The Central Limit Theorem (CLT)

Let $X_1, \ldots, X_n$ be a random sample from a distribution with mean μ and variance σ^2. Then if n is sufficiently large, $\overline{X}$ has approximately a normal distribution with $\mu_{\overline{X}} = \mu$ and $\sigma^2_{\overline{X}} = \sigma^2/n$, and $T_0 = \Sigma X_i$ also has approximately a normal distribution with $\mu_{T_0} = n\mu$ and $\sigma^2_{T_0} = n\sigma^2$. The larger the n, the better the approximation.

100(1-α)% Confidence interval for μ

- SRS, σ known, population normal or large sample size

  $\bar{x} \pm z_{\alpha/2}\, \sigma/\sqrt{n}$
- SRS, population normal or large sample size

  $\bar{x} \pm t_{n-1,\alpha/2}\, s/\sqrt{n}$

100(1-α)% Prediction interval

- SRS, population normal

  $\bar{x} \pm t_{n-1,\alpha/2}\, s\sqrt{1+1/n}$

100(1-α)% Confidence interval for *p*

- SRS and $n\hat{p} \geq 10$, $n(1-\hat{p}) \geq 10$

  $\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}$

Adjusted Wald 95% Confidence interval for *p*

- SRS

  $\tilde{p} \pm 1.96\sqrt{\tilde{p}(1-\tilde{p})/(n+4)}$ where $\tilde{p} = (X+2)/(n+4)$