Winter, 2007

Stat 322 – HW 4 Solutions

1) problem 3 (p. 420), using Table A.9 or Minitab (Calc > Probability Distributions > F) to estimate/determine the p-value. Include a well-labeled sketch of this F distribution with the p-value shaded.

Note, 24 bulbs were tested

We want to test H_0: $\mu_1 = \mu_2 = \mu_3$ vs. H_a: at least one μ_i differs where μ_i is the mean lumen output for brand i

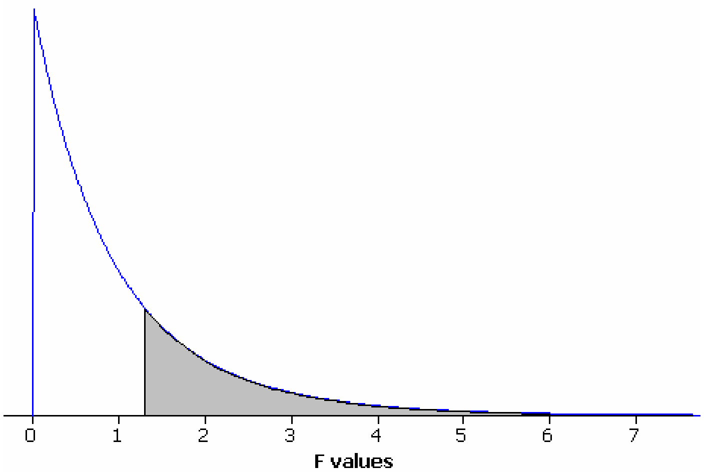
MSE = SSE/(N-I) = 4773.3/(24-3) = 227.3

MSTr = SSTr/(I-1) = 591.2/2 = 295.6

F = MSTr / MSE = 295.6/227.3 = 1.30

with degrees of freedom 2 and 21.

From Table A.9, this F value is smaller than the smallest given (2.57), so we know p-value > .10.




```
F distribution with 2 DF in numerator and 21 DF in denominator

  x   P( X <= x )
1.3     0.706420
```

p-value = 1-.706 = .294, there is a 29.4% chance that we would get an F-statistic this large if there was no difference in the mean lumen output between these 3 brands. So we will reject the null hypothesis (p-value > .05) and conclude that the true average lumen outputs among the three brands for this type of bulb is the same.

2) problem 7 (p. 421) -- **just fill in the table**

Source	df	Sum of Squares	Mean Square	f
Brand	4-1=3	310500.76-235419.04 =75081.72	75081.72/3= 25027.24	25027.24/14713.69 = 1.70
Error	19-3=16	16*14713.69 =235,419.04	14,713.69	
Total	20-1=19	310,500.76		

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (the mean mileages is the same for the four brands)

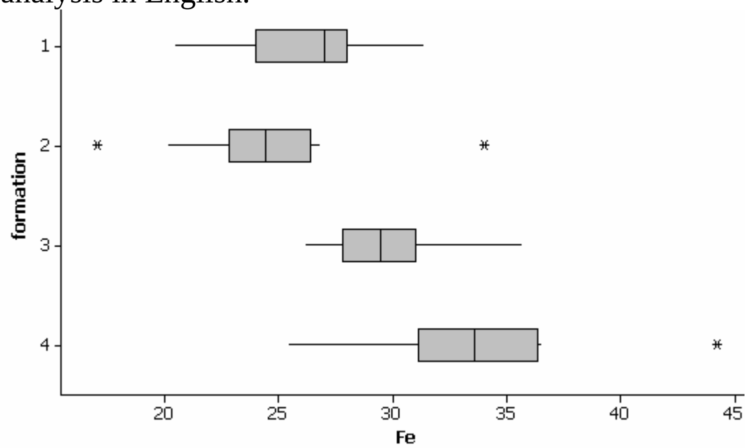
H_a: at least one mean differs

F = 1.70 with degrees of freedom 3 and 16

From the table, we can say p-value > .10 (since 1.70 < 2.46)

We fail to reject the null hypothesis. We do not have evidence that the mean mileage differs among the four brands.

3) Use Minitab to answer problem 6 (p. 421) and problem 15 (p. 427). You should also include boxplots and discussion of technical conditions, and summarize the results of your statistical analysis in English.

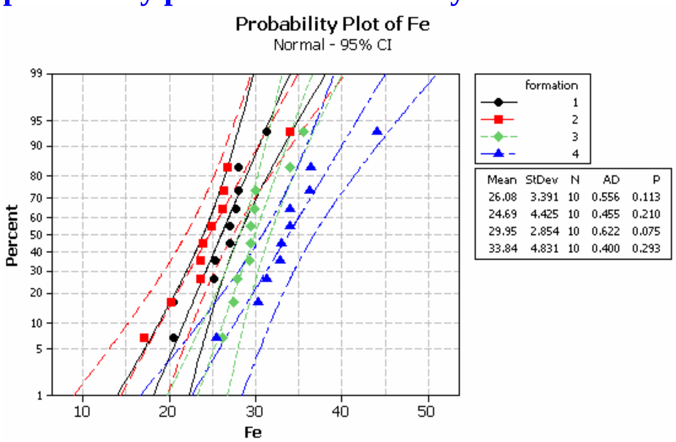


We see evidence that formation 3 and 4 have a tendency for large Fe values. We also see reasons we might be considered about the technical conditions (unequal variances, possible skewness, outliers).

If we look at the summary statistics:

Level	N	Mean	StDev
1	10	26.080	3.391
2	10	24.690	4.425
3	10	29.950	2.854
4	10	33.840	4.831

We will go ahead and consider the equal variance condition met (since the largest SD is not more than twice the size of the smallest SD, 4.831/2.854<2). Also note that the normal probability plots are not the most linear but don't display very serious problems. **Note, the condition is that each of the 4 populations have a normal distribution, so we need to say that each of the 4 probability plots looks reasonably linear.**



Still, it would not be unreasonable to consider transforming the data.

We don't have much information on how they collected the data but it is reasonably to consider the four samples as independent (as long as they didn't forget to clean the machine in between or something).

Using the original data, we will test H_0: $\mu_C = \mu_S = \mu_M = \mu_H$ (the true mean Fe is the same for the four types of iron)
vs. H_a: at least one true mean Fe differs for one of the iron types.

One-way ANOVA: Fe versus formation

```
Source     DF      SS     MS      F      P
formation   3   509.1  169.7  10.85  0.000
Error      36   563.1   15.6
Total      39  1072.3

S = 3.955   R-Sq = 47.48%   R-Sq(adj) = 43.10%
```

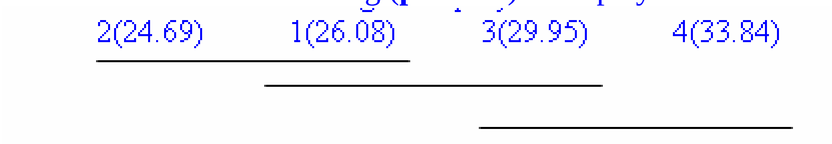
From this output, we would reject the null hypothesis and conclude that there is a difference in the mean iron among the four types of iron formation.

So then we can follow-up with a multiple comparisons procedure.

```
Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons among Levels of formatio

Individual confidence level = 98.93%

formatio = 1 subtracted from:

formatio   Lower  Center   Upper      +---------+---------+---------+--------
2         -6.155  -1.390   3.375                 (------*------)
3         -0.895   3.870   8.635                         (------*-----)
4          2.995   7.760  12.525                              (------*------)
                                      +---------+---------+---------+--------
                                    -14.0      -7.0       0.0       7.0

formatio = 2 subtracted from:

formatio  Lower  Center   Upper     +---------+---------+---------+---------
3         0.495   5.260  10.025                            (------*-----)
4         4.385   9.150  13.915                                 (------*------)
                                    +---------+---------+---------+---------
                                  -14.0      -7.0       0.0       7.0

formatio = 3 subtracted from:

formatio   Lower  Center  Upper     +---------+---------+---------+---------
4         -0.875   3.890  8.655                          (------*-----)
                                    +---------+---------+---------+---------
                                  -14.0      -7.0       0.0       7.0
```

This shows us that formation 1 and 4 differ, formation 2 differs from 3 and 4.

We can use the under scoring **(p. 424-5)** to display this:



From this we could say that formation 3 and 4 (Mag and Hem) appear to differ from 2 (Sil), and 4 (Hem) also differs from 1 (Carb).

4) Use Minitab to answer problem 18 (p. 428)

One-way ANOVA: Hormone, Hormone, Hormone, Hormone, Hormone

```
Source  DF     SS    MS     F      P
Factor   4  200.3  50.1  3.49  0.033
Error   15  215.5  14.4
Total   19  415.8

S = 3.790   R-Sq = 48.17%   R-Sq(adj) = 34.35%
```

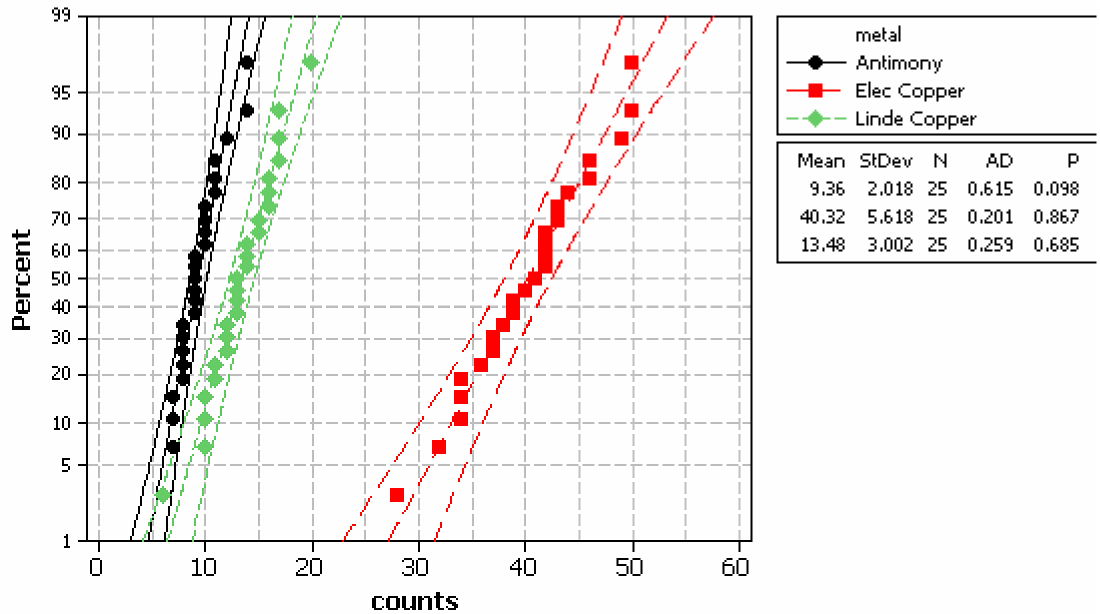
The p-value is statistically significant at the 5% level indicating that at least one population mean (plant growth) differs from the rest. However,

```
Tukey 95% Simultaneous Confidence Intervals
All Pairwise Comparisons

Individual confidence level = 99.25%

Hormone subtracted from:

           Lower  Center   Upper  ------+---------+---------+---------+---
Hormone   -3.282   5.000  13.282              (-------*-------)
Hormone   -3.532   4.750  13.032             (--------*-------)
Hormone   -9.532  -1.250   7.032       (--------*-------)
Hormone  -11.032  -2.750   5.532      (-------*--------)
                                  ------+---------+---------+---------+---
                                      -10         0        10        20

Hormone subtracted from:
           Lower  Center  Upper  ------+---------+---------+---------+---
Hormone   -8.532  -0.250  8.032        (--------*-------)
Hormone  -14.532  -6.250  2.032  (--------*-------)
Hormone  -16.032  -7.750  0.532 (-------*--------)
                                 ------+---------+---------+---------+---
                                     -10         0        10        20

Hormone subtracted from:

           Lower  Center  Upper  ------+---------+---------+---------+---
Hormone  -14.282  -6.000  2.282    (-------*-------)
```

```
Hormone  -15.782  -7.500  0.782  (--------*-------)
                                 ------+---------+---------+---------+---
                                     -10         0        10        20

Hormone subtracted from:

          Lower   Center  Upper  ------+---------+---------+---------+---
Hormone  -9.782  -1.500  6.782        (--------*-------)
                                 ------+---------+---------+---------+---
                                     -10         0        10        20
```

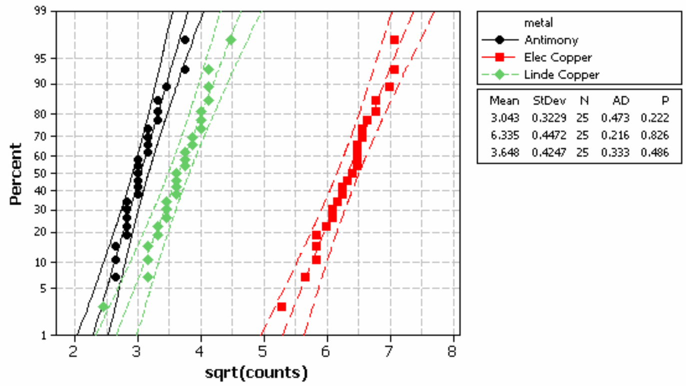
All of the intervals contain zero. “Tukey’s Method and F are at odds.”

5) Porosity of metal, an important determinant of strength and other properties, can be measured by looking at cross sections of the metal under a microscope. The pore (void) is dark, the metal is light and the boundary between these two phases is often clearly delineated. If a grid is laid in the microscopic field, the number of intersections between the grid lines and the pore boundaries is proportional to the length of that boundary per unit area (and hence in proportions to the pore surface area per unit volume of metal). Such counts were made for samples of antimony, Linde copper, and electrolytic copper and are in `porecounts.mtw`.
(a) Check the technical conditions for ANOVA with these data. Which condition appears most seriously violated? Include graphical support for your conclusions.



The linearity of each probability plot indicates that the normality condition is not too unreasonable but the largest standard deviation (5.618) is more than two times the smallest standard deviation (2.018). This indicates that the equal variance condition is not met. We aren’t given much information on how the data were collected but will consider the samples from the 3 metals to be independent.

(b) Apply a square root transformation to the data and see if these data are appropriate for ANOVA? Conduct an ANOVA for these transformed data and state your conclusions.



Normality still seems reasonable and the standard deviations are more similar so we will proceed with the ANOVA.

```
Analysis of Variance for sqrt(counts)

Source  DF       SS      MS       F      P
metal    2  153.498  76.749  475.16  0.000
Error   72   11.630   0.162
Total   74  165.128


S = 0.401898   R-Sq = 92.96%   R-Sq(adj) = 92.76%
```

With such a small p-value, we will reject the null hypothesis and conclude that the mean square root counts are not the same across the three metals.

(c) If discussed in class, why is the square root transformation suggested for these data?
The square root transformation is often applied to count data, especially here where the counts are in proportion to area.