Recap:

- Correspondence between loglinear models and logistic regression models
  - In general, if you have a logistic model, then the corresponding loglinear model includes the highest order interaction involving all of the potential predictors, along with interactions of the predictors with the response variable corresponding to effects in the model.
- Confidence interval for odds ratios using JMP output
- Comparing the model with all two-way interactions vs. model without one of them tests for the homogenous association of two variables conditional on the third
  - Comparison to Cochran-Mantel-Haenszel test

Example 1: A study published in the journal *Pediatrics* (Smith et al., 2006) addressed the important issue of how to awaken children during a house fire so they can escape safely. Researchers worked with a volunteer sample of 24 healthy children aged 6-12 by training them to perform a simulated self-rescue escape procedure when they heard an alarm. Researchers then compared the children’s reactions to two kinds of alarms: a conventional smoke alarm and a personalized recording of the mother’s voice saying the child’s name and urging him or her to wake up. All 24 children were exposed to both kinds of alarms, with the order determined randomly. It turned out that one child did not wake up to either alarm, 14 woke up to both alarms, and 9 woke up to the mother’s voice but not the conventional alarm.

(a) Would the following table be an appropriate way to summarize the data? How can you quickly tell? If not, why not and suggest a better way.

<table>
<thead>
<tr>
<th></th>
<th>Conventional alarm</th>
<th>Mother’s voice</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Awakened</td>
<td>14</td>
<td>23</td>
<td>37</td>
</tr>
<tr>
<td>Did not awaken</td>
<td>10</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>Total</td>
<td>24</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

(b) Calculate the proportion of children that woke up with each alarm. Calculate and interpret an odds ratio for this table.

\[ \hat{\text{proportion}}_{\text{conventional}} = \frac{14}{24} = 0.583 \]

\[ \hat{\text{proportion}}_{\text{mother's voice}} = \frac{23}{24} = 0.958 \]

\[ \text{OR} = \frac{\hat{p}_{\text{mother's voice}} / (1-\hat{p}_{\text{mother's voice}})}{\hat{p}_{\text{conventional}} / (1-\hat{p}_{\text{conventional}})} = \frac{0.583}{0.417} \approx 1.41 \]

The odds of waking up with the mother’s voice are 1.41 times larger than with the conventional alarm.

(c) State the null and alternative hypotheses for testing whether children are more likely to awaken with the mother’s voice. How might you test these hypotheses? Why can’t you?

\[ H_0: \hat{\text{proportion}}_m = \hat{\text{proportion}}_c \]

\[ H_a: \hat{\text{proportion}}_m > \hat{\text{proportion}}_c \]

A two-sample z-test but assumes independent samples.

(d) What are the two variables that we recorded on each child?

- (1) Whether woke up w/ mom’s voice
- (2) Whether woke up w/ conventional alarm
(e) Use the variables in (d) to construct a more appropriate table.

<table>
<thead>
<tr>
<th></th>
<th>Conv= yes</th>
<th>Conv= no</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mom= yes</td>
<td>14</td>
<td>9</td>
<td>23</td>
</tr>
<tr>
<td>Mom= no</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>14</td>
<td>10</td>
<td>24</td>
</tr>
</tbody>
</table>

(f) Compute and interpret the odds ratio for this table.

\[
\text{odds} = \frac{\text{times larger/smaller comparing those who wake up compared to those who don't}}
\]

(g) What notation could I use to refer to the proportions you calculated in (b)? (In terms of \(p\)'s?)

\[
\hat{p}_{\text{Mom}} = \frac{p_{1+}}{n} = \frac{n_{11} + n_{12}}{n}
\]

\[
\hat{p}_{\text{Conv}} = \frac{p_{2+}}{n} = \frac{n_{12} + n_{22}}{n}
\]

(h) What is true about the table in (e) if these proportions are equal? (Hint: How can I calculate each proportion from the entries in the table?)

\[
\text{Need } \frac{n_{12}}{n} \text{ and } \frac{n_{21}}{n} \text{ to be the same}
\]

\[
\frac{n_{12}}{n} = p_{21} \quad \text{and} \quad \frac{n_{21}}{n} = p_{21}
\]

(h) Restate the null and alternative hypotheses for testing the association between these two variables using what you learned in (g). (In terms of \(\pi\)’s?)

\[
H_0: \pi_{12} = \pi_{21}
\]

\[
H_a: \pi_{12} \neq \pi_{21}
\]

(i) What types of tables will constitute strong evidence against the null hypothesis in (h)? (Hint: Does this remind you of any measure of association we have looked at before?)

\[
\text{different values for } n_{12} \neq n_{21}
\]

\[
\text{disconcordant pairs}
\]
If we follow convention and enter these data into the computer with each child as a row and each variable as a column:

<table>
<thead>
<tr>
<th>Child #</th>
<th>Response to mother voice</th>
<th>Response to conventional</th>
<th>Child #</th>
<th>Response to mother voice</th>
<th>Response to conventional</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>wake</td>
<td>wake</td>
<td>13</td>
<td>wake</td>
<td>wake</td>
</tr>
<tr>
<td>2</td>
<td>wake</td>
<td>wake</td>
<td>14</td>
<td>wake</td>
<td>wake</td>
</tr>
<tr>
<td>3</td>
<td>wake</td>
<td>wake</td>
<td>15</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>4</td>
<td>wake</td>
<td>wake</td>
<td>16</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>5</td>
<td>wake</td>
<td>wake</td>
<td>17</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>6</td>
<td>wake</td>
<td>wake</td>
<td>18</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>7</td>
<td>wake</td>
<td>wake</td>
<td>19</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>8</td>
<td>wake</td>
<td>wake</td>
<td>20</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>9</td>
<td>wake</td>
<td>wake</td>
<td>21</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>10</td>
<td>wake</td>
<td>wake</td>
<td>22</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>11</td>
<td>wake</td>
<td>wake</td>
<td>23</td>
<td>wake</td>
<td>not</td>
</tr>
<tr>
<td>12</td>
<td>wake</td>
<td>wake</td>
<td>24</td>
<td>not</td>
<td>not</td>
</tr>
</tbody>
</table>

As always, the big question is: How surprising would the observed experimental result be, under the null model that there’s really no difference in success rates between the two kinds of alarms?

(j) Under the null hypothesis, it doesn’t matter which alarm was going off, the child was going to demonstrate the same behavior. So if we were to model this null hypothesis by flipping a coin to see whether column the two observations are assigned to, what is the “sample size” of this analysis? What is the probability that the responses change places?

The 9 children that differed model as 50/50 which alarm they wake up to

(k) Using the sample size in (j), suggest three different probability models we could use to find a p-value to decide whether there is convincing evidence that the probabilities are not the same. (Hint: Start with an “exact” p-value.)

1. \( X = \# \text{ of children wake up to mom's voice} \)
   \[ \text{mean} = \frac{n_{12} + n_{21}}{2} \]
   \[ \text{p-value} = P(X \geq 9) \]

2. Normal approximation
   \[ \text{mean} = \frac{n_{12} + n_{21}}{2} \]
   \[ \text{SD} = \sqrt{\frac{(n_{12}+n_{21})(15)}{4}} \]
   \[ Z = \frac{n_{12} - \text{mean}}{\text{SD}} = \frac{n_{12} - n_{21}}{\sqrt{n_{12} + n_{21}}/4} = \frac{(n_{12} - n_{21})}{\sqrt{2(n_{12} + n_{21})}} \]

3. Chi-Square:
   \[ \frac{(n_{12} - n_{21})^2}{n_{12} + n_{21}} \]
   McNemar's Test

For our data:

PP: carry out each of these
(l) You can get JMP to calculate this p-value as well. Choose Analyze > Fit Y by X, as with a regular contingency analysis. Specify the two variables as response and explanatory and press OK. Then use the pull-down menu to select Agreement Statistic and look at the p-value for Bowker's Test.

(m) What is a confidence interval for $\pi_{1+} - \pi_{+1}$?