1. With ACE Hardware exterior flat paint, a quart claims to cover 75 square feet. However, tests show some variability. Suppose that the surface area covered by a quart follows a Normal distribution with mean 84.8 square feet and standard deviation 6.4 square feet. (This is based on a large sample of quarts applied to a variety of properly prepared exterior surfaces.)

   a. If we buy one quart, what proportion of quarts will cover at least 75 square feet?

   \[ P(x \geq 75) = P \left( z = \frac{x - \mu}{\sigma} \geq \frac{75 - 84.6}{6.4} = -1.53 \right) = 1 - 0.0630 = 0.9370 \]

   b. What is the coverage that we can be 99% sure of obtaining? (i.e. how many square feet will one be able to cover with 99% of the quarts?)

   If we find the 1st percentile of coverage, 99% of the quarts will cover at least that amount. \( z = -2.33, \) so \( x = \mu + z \sigma = 84.8 + (-2.33) \times 6.4 = 69.9 \) square feet.

Suppose now that that instead of Normal, the distribution of the amount covered were to follow a Uniform distribution (with equal chance for each possible value) over the range 73.71 square feet to 95.89 square feet (which has mean 84.8 and SD 6.4). i.e. \( f(x) \) is a constant for 73.71 \( \leq x \leq 95.89 \) and zero elsewhere.

   c. Determine the chance that one sampled quart will cover 75 square feet.

   For uniform on the range 73.71 to 95.89, the height of our density is \( 1/(95.89-73.71) = 0.0451. \) Thus the chance that we can cover at least 75 square feet is the area of the rectangle of height .0451 that runs from 75 to 95.89 which is .9418.

   \[ \text{[Some might prefer the more mathy version: } P(X > 75) = \int_{75}^{95.89} \frac{1}{95.89 - 73.71} \, dx = \frac{95.89 - 75}{95.89 - 73.71} = 0.9418 \] \]

   Bonus: What is the coverage that we can be 99% sure of obtaining?

   Because this uniform distribution is 22.18 wide (running from 73.71 to 95.89), the 1st percentile would be 1/100th of the way from 73.71 to 95.89. Thus, 99% of the time we will be able to cover at least 73.93 square feet.