Things that one should feel comfortable with:

1 – The backshift operator
2 – the ARMA model
   a – in regular form
   b – in the form of the backshift operator
3 – Differencing
4 – ACF/PACF of standard AR and MA models
5 – Use of Yule-Walker to determine the ACF of the AR model
6 – Derivation of the ACF of the MA model
7 – forecasting (conceptually)
8 – deriving the variance of the forecast errors
10 – determining stationarity of ARMA models (the unit circle approach)
11 – converting an ARMA(p,q) model into an AR(∞) model
12 – converting an ARMA(p,q) model into an MA(∞) model
   i.e. find coefficients, ψ₁, ψ₂, ψ₃, etc. of \( \Psi(B) = \frac{\Theta(B)}{\Phi(B)} \) where
   \[ \Psi(B) = 1 - \psi₁B - \psi₂B² - \psi₃B³ - \cdots, \]
   \[ \Theta(B) = 1 - \theta₁B - \theta₂B² - \theta₃B³ - \cdots - \theta_qB^q, \]
   \[ \Phi(B) = 1 - \phi₁B - \phi₂B² - \phi₃B³ - \cdots - \phi_pB^p. \]
13 – recognizing from the ACF/PACF combination the nature of the model that could
   or should be fit

Hypothetical questions:

1. For each of the following pairs of ACF and PACF graphs, identify the model as AR(1),
   AR(2), MA(1), MA(2) or ARMA(1,1) and explain which aspects of which graphs
   determined your choice...
2. Explain what characteristics of a pair of ACF and PACF graphs would suggest that you should take the 1st difference of the data before analysis.

3. For an AR(2) model:
   a. Use the Yule-Walker approach to calculate the theoretical ACF.
   b. Determine whether the pair \( \phi_1 = 1.5 \) and \( \phi_2 = -0.9 \) provides a stationary or non-stationary process.
   c. The pair \( \phi_1 = 1.5 \) and \( \phi_2 = -0.9 \) results in an ACF that is “decaying” and “oscillatory” in nature. Why is this be the case?

4. Derive forecasts and approximate forecast error variances for an MA(2) model for forecast horizons of length 1, 2 and 3.

5. Question 5
   a. Write the ARIMA(0,2,2) model in the standard form with the backshift operator, and if necessary, polynomial functions of the backshift operator, \( \Phi(B) \) and \( \Theta(B) \). Be sure to define \( \Phi(B) \) and \( \Theta(B) \) if you use either.
   b. If we were to rewrite this as \( x_t = \Psi(B)e_t \), what is \( \Psi(B) \) equal to in terms of \( \Phi(B), \Theta(B) \), etc.? (In other words, do some algebra on your answer in part to determine \( \Psi(B) \) in an convenient form.)
   c. If \( \Psi(B) = 1 - \psi_1 B - \psi_2 B^2 - \psi_3 B^3 - \ldots \), determine \( \psi_1, \psi_2 \) and \( \psi_3 \).
   d. Determine approximate forecast variances for a forecast horizon of 4.

6. Derive the:
   a. variance of:
      i. An AR(1) process
      ii. A seasonally autoregressive process of the first order.
   b. Autocorrelation function for a SARIMA(0,0,1) \( (0,0,2) \) process.

7. Express the seasonal ARMA model: \( (1 - \phi_1 B)x_t = (1 - \theta_{12} B^{12})e_t \) in the form \( x_t = \Psi(B)e_t \). I.e., determine \( \Psi(B) \).

8. With the seasonal ARMA model \( (1 - \phi_{12} B^{12})x_t = (1 - \theta_3 B)e_t \), [note: this model differs from that of the previous question] what do you think the ACF of this model would look like? Draw a picture and explain.

9. Suppose you believe your time series to be ARIMA(p,d,q) for some p, d and q. Explain how you could determine p, d and q.
1. Sketch the ACF and PACF of an AR(1) model with $\phi_1$ positive and large.
2. Sketch the ACF and PACF of an MA(1) model with $\theta_1$ negative and large.
3. Sketch the ACF and PACF of an MA(2) model with $\theta_1$ positive and large and $\theta_2$ negative but not as large.
4. Sketch the ACF and PACF of a seasonally autoregressive model with $\Phi_1$ negative but neither large nor small (think: $-0.6 < \Phi_1 < -0.4$ or so).
5. For an ARIMA(0,1,1) model $(1-B)y_t = (1-\theta_1)\varepsilon_t$ where $\varepsilon_t$ is normal white noise with variance $\sigma^2$.
   a. Write out the model in standard form
   b. Suppose you want to forecast $y_{t+1}$ based on all the data available through time $t$.
      Determine your forecast. Be sure to state in terms of only current and past $y$ values and residuals.
   c. Suppose you want to forecast $y_{t+2}$ based on all the data available through time $t$.
      Determine your forecast. Be sure to state in terms of only current and past $y$ values and residuals.

Now we need to determine forecast errors
   d. Rewrite your model as $y_t = \Psi(B)\varepsilon_t$, where $\Psi(B) = \frac{(1-\theta_1)}{(1-B)} = (1 - \psi_1 B - \psi_2 B^2 - \cdots)$. I.e. determine the coefficients $\psi_1, \psi_2, \psi_3, \ldots$.
   e. What is the one-step ahead forecast error variance?
   f. What is the two-step ahead forecast error variance?
   g. What is the $k$-step ahead forecast error variance?