I. Some conceptual/theoretical problems

1. Suppose that we use single exponential smoothing but choose an initial level \( L_0 = -100 \). Suppose also that each and every data value \( y_1=y_2=\ldots=1 \). With alpha = .2, at what time period would the level be at least positive?

2. For double exponential smoothing (Holt’s linear trend),
   a. Use algebra to rewrite the model such that \( L_t = a_0 y_t + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + \ldots \) [i.e. determine the weights, \( a_0, a_1, a_2, a_3, \text{ etc.} \)]
   b. Supposing an increasing linear trend. Will any of the weights (the \( a \)'s) be negative? Explain why or why not.

II. Some more conceptual/theoretical problems

1. If \( y_t = \beta_0 + \beta_1 t + \epsilon_t \) and the \( \epsilon_t \) are independent of each other mean zero random variables:
   a. Explain why a centered moving average of length \( k \) would be an unbiased estimate of the linear trend. [Hint: \( E(\epsilon_t) = 0 \).]
   b. Explain why a realizable moving average of length \( k \) would be a biased estimate of the linear trend. [Hint: \( E(\epsilon_t) = 0 \).]
   c. Determine whether Holt’s linear trend (double exponential smoothing) prediction of \( y_{t+1} \) with \( L_{t+1} \) would be biased or not.

2. If we have a linear trend plus seasonality model: \( y_t = \beta_0 + \beta_1 t + \beta_{\text{Jan}} I_{\text{Jan},t} + \beta_{\text{Feb}} I_{\text{Feb},t} + \ldots + \beta_{\text{Nov}} I_{\text{Nov},t} + \epsilon_t \) and the \( \epsilon_t \) are independent of each other mean zero random variables and \( I_{\text{Month},t} \) is a month indicator, show that a 12 period moving average (whether central or realizable) would not have any seasonality.

III. Some data oriented problems

1. For Beer production (R code to read this in is provided):
   b. Using an appropriate exponential smoothing method, determine good values for the smoothing parameters (alpha, beta, gamma).
   c. Explain why the chosen values are appropriate based on just the plot of the data.
   d. Forecast the next three years of the series and plot alongside the last 6 years of the actual series.

2. Repeat a-d for the last 10 years of the Global Temperature dataset.

IV. Yet another conceptual problem.

1. Suppose that there is a linear trend and seasonality. Which of regression, Holt-Winters seasonal exponential smoothing and moving average methods would be the most appropriate for estimating the trend. Explain.

2. Again supposing that there is a non-linear trend and seasonality, which of the three methods, regression, Holt-Winters seasonal exponential smoothing and moving average methods would be most appropriate for estimating the trend. Explain.