Time Series: Autoregressive Models

AR(1)

1. Create an AR(1) series with $\phi_1=.7$. Describe the ACF and the PACF, paying special attention to the number of distinct spikes, the sign of those spikes and the rate of decay to zero.

2. Repeat with $\phi_1=.4$. How do the ACF and PACF differ?

3. Repeat with $\phi_1=.95$. How do the ACF and PACF seem to respond to the size of $\phi_1$?

4. Repeat with $\phi_1 = -.7$. Describe the ACF and PACF now, again paying special attention to the number of distinct spikes, the signs of those spikes and rate of decay to zero.

5. What do you expect to see, if $\phi_1 = -.3$. Write it down and try it. Was your guess correct? Did anything surprise you?

AR(2)

6. Create an AR(2) series with $\phi_1=.6$ and $\phi_2=.35$. Plot the series along with the ACF and PACF. Describe all three.
7. Repeat for $\phi_1 = .6$ and $\phi_2 = -.35$.

8. $\phi_1 = -.6$ and $\phi_2 = .35$

9. $\phi_1 = -.6$ and $\phi_2 = .35$

10. Use the Yule-Walker approach to verify your ACF values.

11. Now for an AR(2) series with $\phi_1 = -.9$ and $\phi_2 = -.85$ create the series, plot it then look at the ACF/PACF. Describe.

12. Same, but for $\phi_1 = .9$ and $\phi_2 = -.85$. 
13. What would you expect the PACF of an AR(3) model would look like?

14. What would you expect the ACF of such an AR(3) model would look like? Use the Yule-Walker approach to find the ACF values for $\phi_1 = .5$ and $\phi_2 = .3$ and $\phi_3 = -.2$.

15. Generate a series with $\phi_1 = .5$ and $\phi_2 = .3$ and $\phi_3 = -.2$ and verify your above answers. Were you right? Explain.

16. If a series is an AR(p) it will have a PACF that ...

17. So we can determine the order of the autoregressive model by ...

Note: If the ACF doesn’t decay to zero in what could be described as an exponential fashion (perhaps with oscillation) it probably is not an autoregressive process.